Exam 1 Review (Section 004)

$\boxed{\#28}$ $h(t) = \dfrac{3t-1}{\sqrt{t}-2}$, find $h'(16)$.

$\frac{d}{dt}[\sqrt{t}-2] = \frac{d}{dt}[t^{1/2}-2]$
$= \frac{1}{2}t^{-1/2}$

$$h'(t) = \frac{(\sqrt{t}-2)(3) - (3t-1)(\frac{1}{2}t^{-1/2})}{(\sqrt{t}-2)^2}$$

$$h'(16) = \frac{(\sqrt{16}-2)(3) - (3\cdot 16-1)(\frac{1}{2}(16)^{-1/2})}{(\sqrt{16}-2)^2}$$

$$= \frac{(4-2)(3) - (3\cdot 16-1)(\frac{1}{2}\cdot\frac{1}{4})}{(4-2)^2}$$

$$= \frac{2\cdot 3 - (3\cdot 16-1)(\frac{1}{8})}{2^2}$$

$\boxed{\#39}$ $g(x) = \dfrac{2\sqrt{x}}{x^2-8} = \dfrac{2x^{1/2}}{x^2-8}$

$$g'(x) = \frac{(x^2-8)(\cancel{2}(\cancel{\tfrac{1}{2}}x^{-1/2})) - 2\sqrt{x}(2x)}{(x^2-8)^2}$$

$\boxed{Ex}$ $f(x) = \underline{x}\,\underline{e^x \tan x}$ (You could also choose the factors differently!)

$f'(x) = 1(e^x \tan x) + x\frac{d}{dx}[\underbrace{e^x \tan x}_{\uparrow\ \text{PRODUCT!}}]$

$= e^x \tan x + x\,[e^x \tan x + e^x \cdot \sec^2 x]$

$$\boxed{\begin{aligned} &= e^x \tan x + xe^x \tan x + xe^x \sec^2 x \\ &= e^x(\tan x + x\tan x + x\sec^2 x)\end{aligned}}$$

$\boxed{\#26}$ 240-ft building

$s(t) = -16t^2 + 32t + 240$ (height)

what is $v(t)$ when the ball hits the ground?

↳ Find where $s(t) = 0$

$$\frac{-16t^2 + 32t + 240 = 0}{-16}$$

$$t^2 - 2t - 15 = 0$$

$$(t-5)(t+3) = 0$$

$$t = 5 \text{ or } \cancel{t = -3}$$

Then find $v(5)$.

$\boxed{\text{HW 9 \#10}}$

$y = \underbrace{4x}\underbrace{\sin x}$. Find tangent line at $x = \pi$.

$y' = 4\sin x + 4x\cos x$

$y(\pi) = \cancel{4\pi \sin\pi} \to 0$

$y'(\pi) = \cancel{4\sin\pi}^{\,0} + 4\pi\cos\pi$

$m = -4\pi$

$(\pi, 0) = (x_0, y_0)$

$$y - y_0 = m(x - x_0)$$

$$y - 0 = -4\pi(x - \pi)$$

$$y = -4\pi x + 4\pi^2$$

(Eqn of circle: $(x-h)^2 + (y-k)^2 = r^2$)

OR $y = mx + b$ at $(\pi, 0)$: $0 = -4\pi(\pi) + b$

$y = -4\pi x + b$ $\qquad b = 4\pi^2$

$\boxed{\text{HW 10 \#13}}$ (Other Functions #3)

$$y = \underbrace{5x^9}\underbrace{\csc x}$$

$$y' = 5(9x^8)\csc x + 5x^9(-\csc x \cot x)$$

$$= \boxed{\begin{aligned} &45x^8 \csc x - 5x^9 \csc x \cot x \\ =\ &5x^8 \csc x\,(9 - x\cot x) \end{aligned}}$$

$\boxed{\#2}$

$\lim_{x\to1} f(x) \neq f(1)$ exists

I. True ($x=1$ hole, $x=2$ jump, $x=4$ hole)

II. False ($\lim_{x\to2^-} f(x) = 2$, $\lim_{x\to2^+} f(x) = 3$)

III. False ($\lim_{x\to4^-} f(x) = 2 = \lim_{x\to4^+} f(x)$)

IV. False ($\lim_{x\to1} f(x) = 1$)

Exam 1 Review (section 023)

#5 $P(t) = 10 + \frac{50t}{2t^2+9}$

Find $P'(2)$

$$P'(t) = \frac{(2t^2+9)(50) - 50t(4t)}{(2t^2+9)^2}$$

$$P'(2) = \frac{(2\cdot 2^2+9)(50) - 50(2)(4)(2)}{(2\cdot 2^2+9)^2}$$

$$= \boxed{0.173}$$

#7 $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{\frac{(x+h)^2}{2} - \frac{x^2}{2}}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{2} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{2h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{2h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{2\cancel{h}}$$

$$= \lim_{h\to 0} \frac{2x+h}{2}$$ ← mistake in line (4)

#35 $f(x) = 6\sin x$. Find tangent line at $x = \frac{\pi}{3}$.

$f'(x) = 6\cos x$

$f(\frac{\pi}{3}) = 6\sin(\frac{\pi}{3})$

$= 6(\frac{\sqrt{3}}{2})$

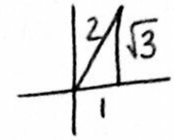
$= 3\sqrt{3}$

$(\frac{\pi}{3}, 3\sqrt{3}) = (x_0, y_0)$

$f'(x) = 6\cos(\frac{\pi}{3})$

$= 6(\frac{1}{2})$

$= 3$



$m = 3$

$y - y_0 = m(x - x_0)$

$y - 3\sqrt{3} = 3(x - \frac{\pi}{3})$

$\boxed{y = 3x - \pi + 3\sqrt{3}}$

OR

$y = mx + b$

$y = 3x + b$

$3\sqrt{3} = 3(\frac{\pi}{3}) + b$

$3\sqrt{3} - \pi = b$

#40 $y = (\tan x)(\sec x + 1)$

$y' = \sec^2 x\,(\sec x + 1) + \tan x\,(\sec x \tan x)$

$= \boxed{\sec^3 x + \sec^2 x + \sec x \tan^2 x}$

$= \boxed{\sec x\,(\sec^2 x + \sec x + \tan^2 x)}$

#4 Which is not $+\infty$?

- $\lim_{x \to 2^-} \frac{x+2}{x-2} = \frac{4}{-\text{SMALL}} = -\infty$ ✓
- $\lim_{x \to 3^+} \frac{x}{\sqrt{x^2-9}} = \frac{3}{+\text{SMALL}} = +\infty$ ✗
- $\lim_{x \to 1^+} \frac{1}{x-1} = \frac{1}{+\text{SMALL}} = +\infty$ ✗

#8 $f(x) = \frac{1}{x+1}$, $g(x) = \frac{x-1}{x^2-1}$

- $\lim_{x \to 1} g(x) = \lim_{x \to 1} \frac{x-1}{x^2-1} = \frac{0}{0}$ ← something cancels

$$= \lim_{x \to 1} \frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}}$$

$$= \lim_{x \to 1} \frac{1}{x+1} = \frac{1}{2}$$

so the limit exists!

- $\lim_{x \to -1} \frac{1}{x+1} = \frac{1}{0}$ ← VA

$$\left.\begin{aligned} &\lim_{x \to -1^-} \frac{1}{x+1} = \frac{1}{-\text{SMALL}} = -\infty \\ &\lim_{x \to -1^+} \frac{1}{x+1} = \frac{1}{+\text{SMALL}} = +\infty. \end{aligned}\right\} \text{limit DNE}$$

- $\lim_{x \to -1} \frac{x-1}{x^2-1} = \frac{-2}{0}$ ← VA

$$\left.\begin{aligned} &\lim_{x \to -1^-} \frac{x-1}{x^2-1} = \frac{-2}{+\text{SMALL}} = -\infty \\ &\lim_{x \to -1^+} \frac{x-1}{x^2-1} = \frac{-2}{-\text{SMALL}} = +\infty \end{aligned}\right\} \text{limit DNE}$$